References

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Drop Breakup and Liquid Jet Penetration

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THE transient response of a liquid drop to a suddenly applied aerodynamic flowfield has been studied extensively in shock tubes by establishing a falling column of drops in the driven section and observing their behavior after shock passage. Reference 1 provides a summary of such work and lists relevant references. The behavior of a liquid jet steadily injected into a gas stream also has received extensive study in supersonic wind tunnels, where the jet is injected normal to the gas flow from the side wall or from a flat plate aligned with the stream. Reference 2 presents the results of such experiments and a list of references. The purpose of this Note is to show the phenomenological and quantitative similarity of drop shattering and liquid jet penetration.

Experimental observations of drop behavior after shock passage have included the time t_d for the drop to shatter and its drag-induced displacement x from the initial location. These quantities generally have been nondimensionalized by the diameter D and density ρ_d of the drop and by the air density ρ_2 and speed U_2 behind the shock wave and relative to the undisturbed drops, resulting in the dimensionless quantities

$$X = x/D \tag{1}$$

$$T_d = t_d U_2 \sqrt{\rho_2/\rho_d}/D \tag{2}$$

The dimensionless breakup time defined in this way has been shown to depend primarily, but weakly, on the Weber number

$$We = D\rho_2 U_2^2/\sigma \tag{3}$$

where σ is the drop surface tension.[1] Specifically, T_d varies as $We^{-1/4}$, a fact that indicates that a drop is shattered by the growth of unstable Rayleigh-Taylor waves on its front surface.[1,3] For Weber numbers greater than about 500, T_d is of order 4.[1] The downstream displacement of the drop commonly is correlated for engineering purposes in the form

$$X = AT^2 \tag{4}$$

where A varies from 0.5 to 1.4 for Weber numbers above 300 and depends on the Weber number and the time interval observed after shock passage.[1,4]

Turning to the phenomenon of liquid jet penetration into a gas flow, the experimental observations include the height h of jet penetration and the coordinates x, y of the jet.[2,5] The penetration commonly is nondimensionalized and correlated as

$$hM/D = K\sqrt{p_j^0/p} \tag{5}$$

where D is the jet initial (or orifice) diameter, M and p are the freestream Mach number and static pressure, and p_j^0 is the jet plenum pressure. Since the liquid jet is usually underexpanded, Eq. (5) also can be written as

$$h/D = K\sqrt{\gamma \rho_j U_j^2/2\rho U^2} \tag{6}$$

where $\rho_j U_j^2$ and ρU^2 are the jet and airstream dynamic pressures, respectively, and γ is the specific heat ratio of the air. Typical values of K were about 7 in experiments where the Weber number was generally above 500.[2,5]

The breakup of the liquid jet is similar to the breakup of a drop in that, upon emerging from the jet orifice, an element of the jet is, like a drop overtaken by a shock, suddenly subjected to aerodynamic forces tending to accelerate and shatter it. Assuming that the jet element moves normal to the air velocity at constant speed U_j, the distance h that the jet penetrates into the flow is the product of the time t_j required to shatter the jet element times the speed of the jet, that is, $h = t_j U_j$. From this relation and from Eq. (6), we can derive the dimensionless time for jet breakup:

$$T_j = t_j U \sqrt{\rho/\rho_j}/D = 5.85 \quad (\text{for } \gamma = 1.4,\ K = 7) \tag{7}$$

Comparison of this result with Eq. (2) shows that the forms of the nondimensional times are identical. Moreover, the values of the dimensionless times T_d and T_j are quite close, about 4 and 6, respectively.

We also can compare the downstream displacement of the jet element with that of the drop, since both motions result from aerodynamic drag. Values of the downstream displacement of the jet element as a function of its time after injection y/U_j, where x and y are the coordinates of the jet measured from the orifice parallel and normal to the freestream velocity, respectively, are available in Ref. 5. A plot of these data in the nondimensional form of $X = x/D$ as a function of $T = (y/U_j)(U/D)\sqrt{\rho/\rho_j}$ is shown in Fig. 1, together with a parabolic least-squares regression:

$$X = 0.8T^2, \quad T \leq 6 \tag{8}$$

Comparison of this result with Eq. (4) again indicates both the similarity of the dimensionless groups and correlation and the agreement of the empirical coefficients. This agreement results not only from the physical similarity of the jet and drop shattering phenomena, but also from the fact that in

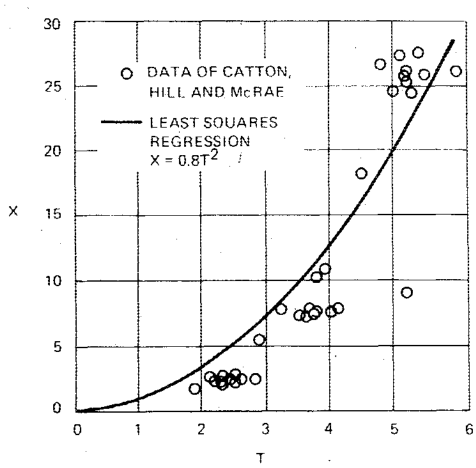

Fig. 1 Dimensionless downstream displacement of a liquid jet.

Received Nov. 2, 1977; revision received March 10, 1978.

Index categories: Jets, Wakes, and Viscous-Inviscid Flow Interactions; Multiphase Flows.

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supersonic flow the ballistic coefficients of a spherical drop and a cylindrical element are nearly equal.

We conclude on the basis of this discussion that the fundamental similarity of liquid jet penetration to drop breakup is established quantitatively. Other similarities probably can be found. For example, if the shattering results from the consumption of liquid mass by the unstable growth of surface waves,[1] then the breakup time of a liquid element should be proportional to the volume-to-wetted-area ratio of the element. This ratio has the proportions of 2 to 3 for a sphere with respect to a cylindrical element, in good agreement with the ratio of respective breakup times, which are about 4 to 6, respectively. As another example, the jet penetration regimes defined in Ref. 2 may correspond to modes of drop breakup identified in Ref. 1. Apparent dissimilarities also exist, however. An important example is the role played by surface tension. Both theory and experiment indicate that drop breakup has a weak dependence on surface tension,[1,3] while in Ref. 5 it is concluded on the basis of experimental data that surface tension has no influence on jet penetration.

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Multiburst Cloud Rise

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Nomenclature

a = cloud aspect ratio $(2b/\Delta h)$
b = cloud horizontal radius
F = $(2\pi b^3/a)g[(\rho_0-\rho)/\rho_I]$
G = $-(g/\rho_I)(\mathrm{d}\rho_0/\mathrm{d}x)$
g = acceleration due to gravity
H = stabilization altitude of cloud
Δh = cloud thickness
N = number of bursts
R = nondimensional cloud radius (see Table 1)
s = burst separation
t = time
t_I = nondimensional time (see Table 1)
U = nondimensional vertical velocity (see Table 1)
u = mean vertical velocity of cloud
W = nuclear weapon yield
x = altitude
X = nondimensional altitude (see Table 1)
α = entrainment constant
β = $b_0^2/N(s^2/4)$
Δ = nondimensional cloud buoyancy (see Table 1)
ρ = average density inside cloud
ρ_0 = ambient density at cloud level
ρ_I = ambient density at source level

Subscripts

ES = extended source
MB = multiburst
0 = initial value, except ρ_0
PS = point source
SB = single burst

Received Nov. 10, 1977.
Index category: Hydrodynamics.
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Introduction

THE dimensions and stabilization altitude of the cloud produced by a near-surface nuclear burst are of interest to the weapons system designer. The gross motion of the cloud, except at very early times, is like that of a buoyant thermal, and solutions[1-3] are available for a single burst. Equivalent models are not available for the case where two or more rising clouds interact. It is the purpose of this Note to present an extension of the instantaneous point source solution of Ref. 1 to a limiting case of many bursts which are closely spaced and nearly simultaneous.

Morton et al.[1] obtained a closed-form solution for a uniformly and stably stratified fluid. More general one-dimensional finite-difference cloud-rise codes which utilize the same basic assumptions, have been used in nuclear cloud-rise calculations. The basic assumptions are: 1) an incompressible fluid, except that small differences in (potential) density are allowed in buoyancy terms (Boussinesq approximation), and 2) the Taylor[4] entrainment hypothesis. One example of such a code is the cloud-rise module[5] of the DOD Land Fallout Prediction System—DELFIC. The density differences due to a nuclear burst are not initially small. This problem is avoided by choosing the initial time for cloud rise to be after the pressure in the fireball has returned to ambient. Existing nuclear burst data have been used[6] to determine initial conditions as a function of weapon yield and height of burst.

Analysis

Consider an initial cloud in the form of a circular disk of diameter $2b_0$ and thickness $\Delta h_0 = 2b_0/a$. The following assumptions are made:

1) The fluid is incompressible, except that small $[(\Delta\rho/\rho) \ll 1]$ variations in density are allowed in buoyancy terms.

2) Profiles of velocity and buoyancy through the cloud have the same form.

3) The rate of entrainment of ambient air at the cloud boundary is proportional to the mean vertical velocity of the cloud.

4) The entraining surface is taken to be the two faces of the disk. The rim area is small and the analysis is simpler without it.

With these assumptions, conservation of mass, momentum and energy (sometimes referred to as conservation of density deficiency or buoyancy) can be written

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{2\pi b^3}{a}\right) = 2\pi b^2 \alpha u \qquad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{2\pi b^3}{a}\rho u\right) = \left(\frac{2\pi b^3}{a}\right) g(\rho_0-\rho) \qquad (2)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{2\pi b^3}{a}(\rho_I-\rho)\right] = 2\pi b^2 \alpha u(\rho_I-\rho_0) \qquad (3)$$

and a fourth equation is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u \qquad (4)$$